Empirical Characterization of Discretization Error in Gradient-based Algorithms

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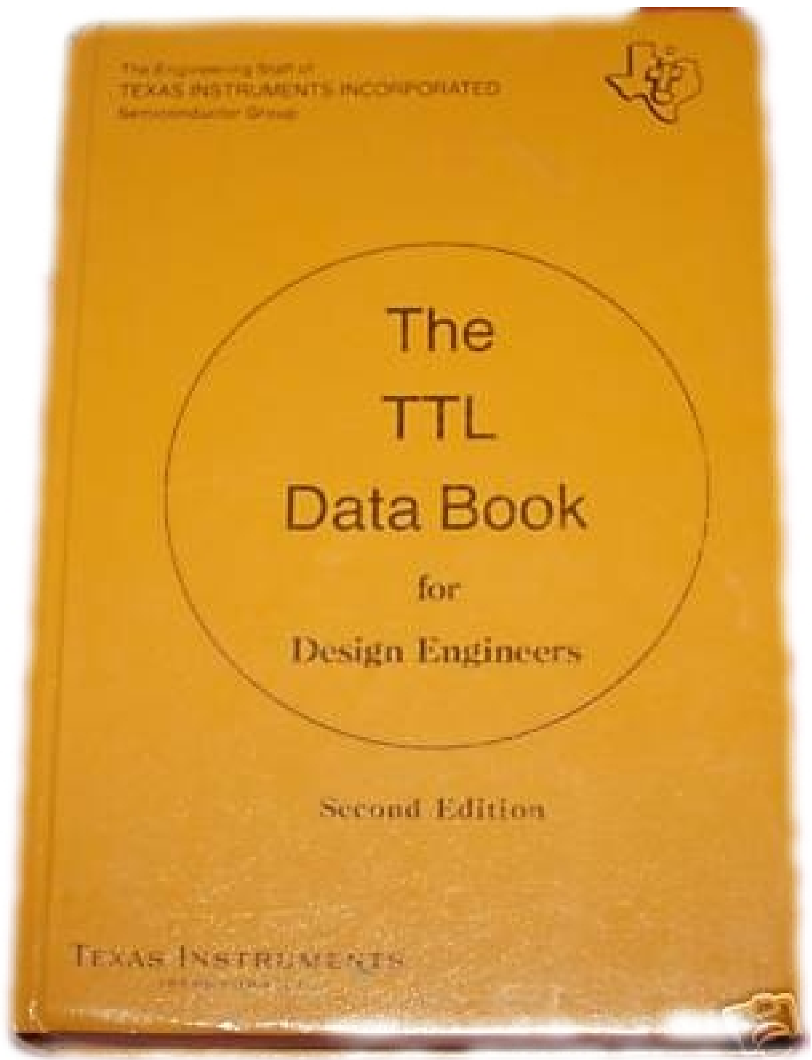
BBN TECHNOLOGIES

The Challenge of Composition

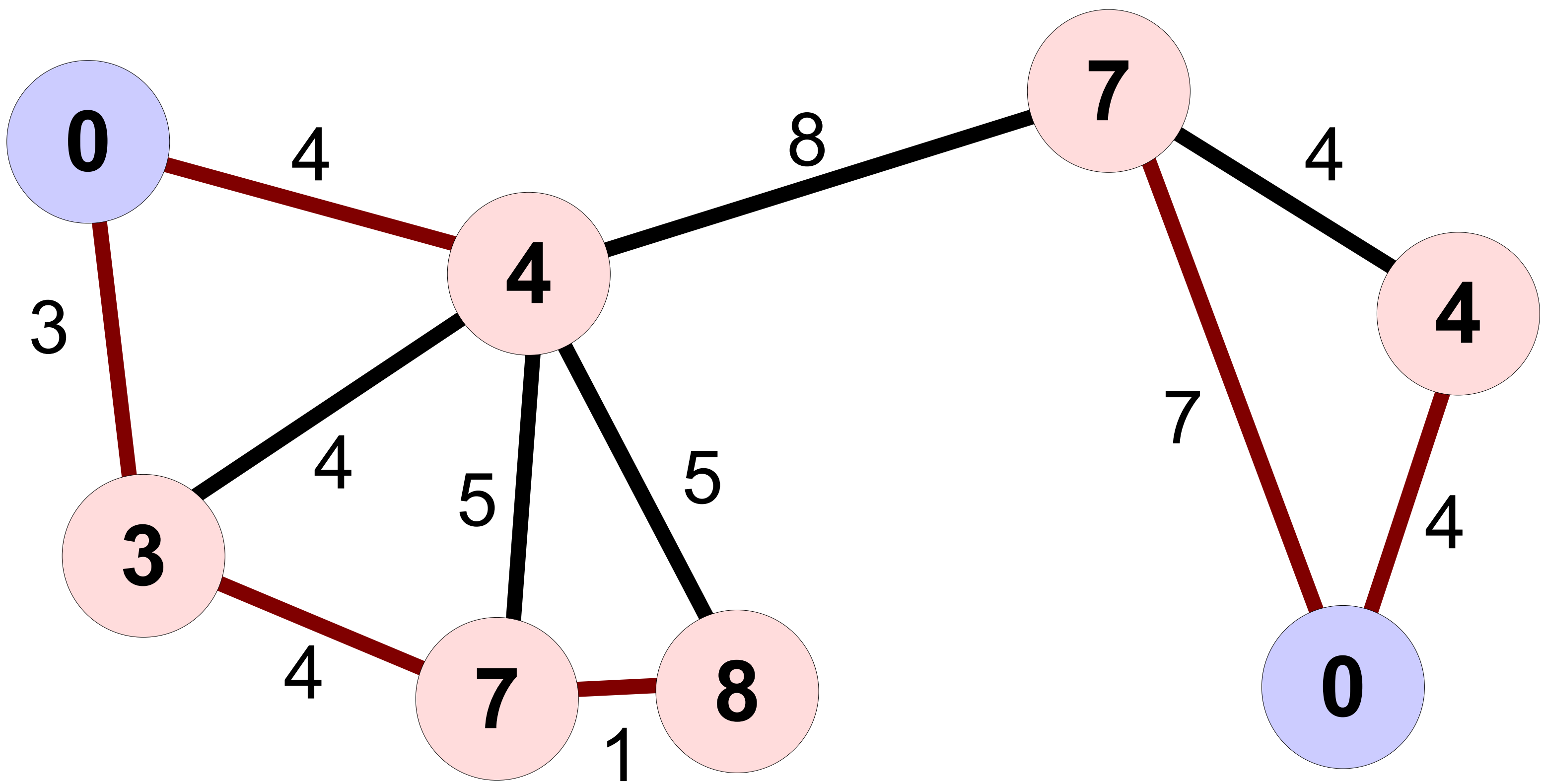


Outline

- Gradients
- Discretization Error
- Empirical Model
- Predictive Composition

Gradient

Distance from each device to nearest source



Distance in graph is proxy for real distance

Geometric Program: Bisector

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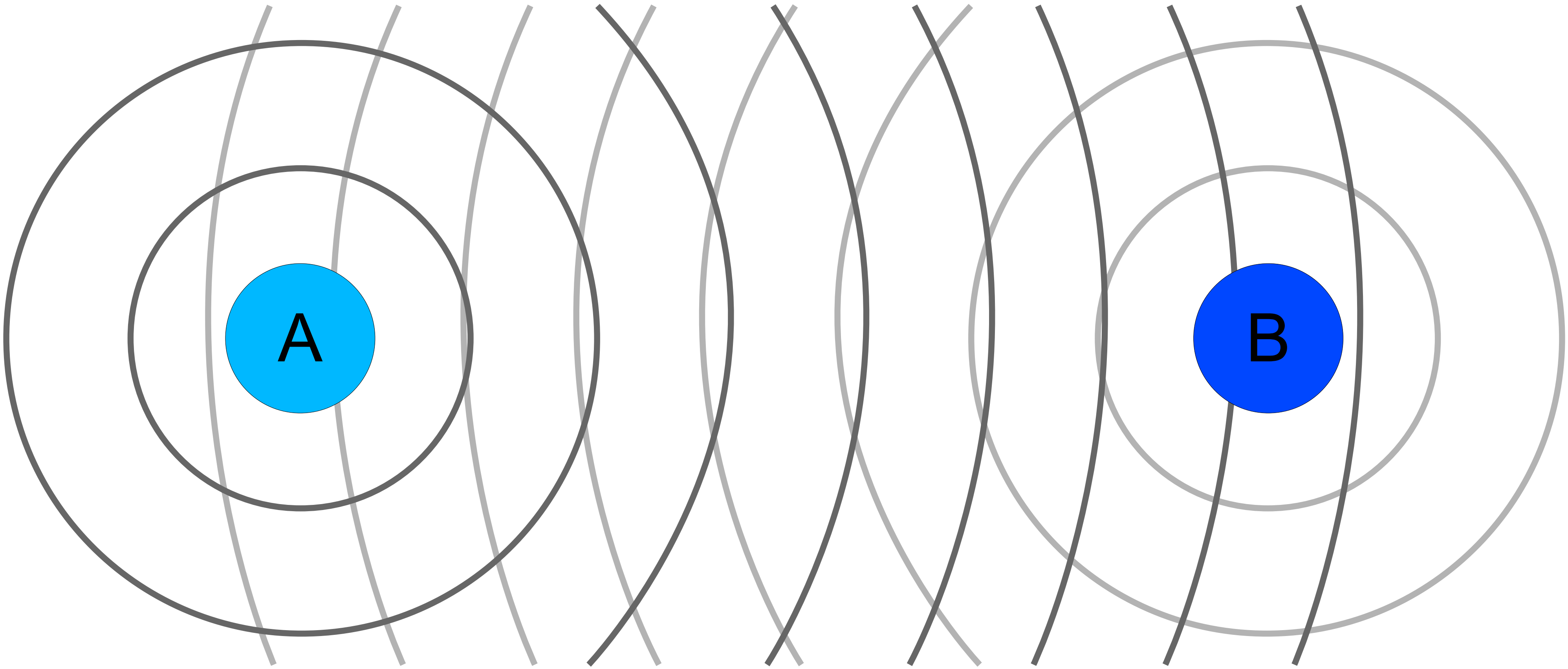
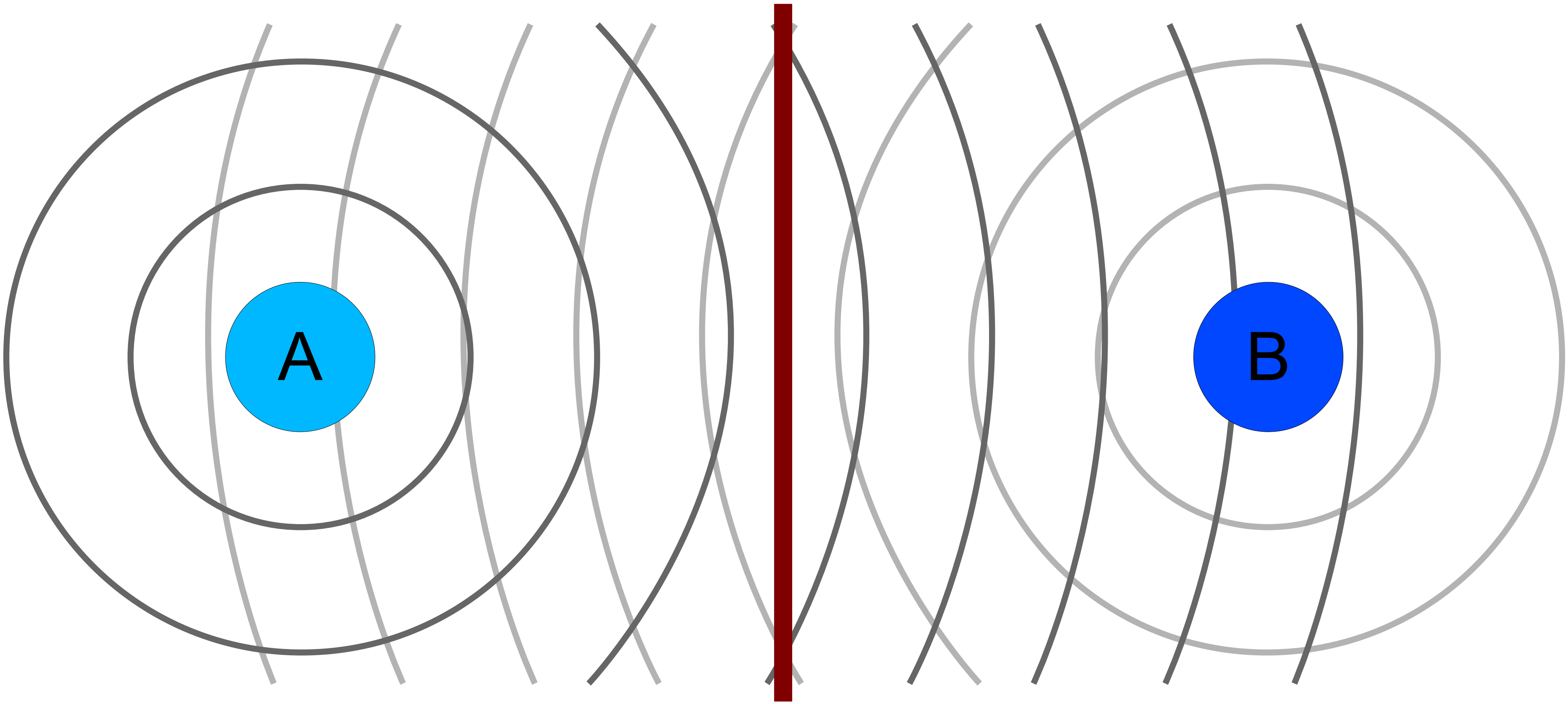


Geometric Program: Bisector



Geometric Program: Channel

(cf. Butera)

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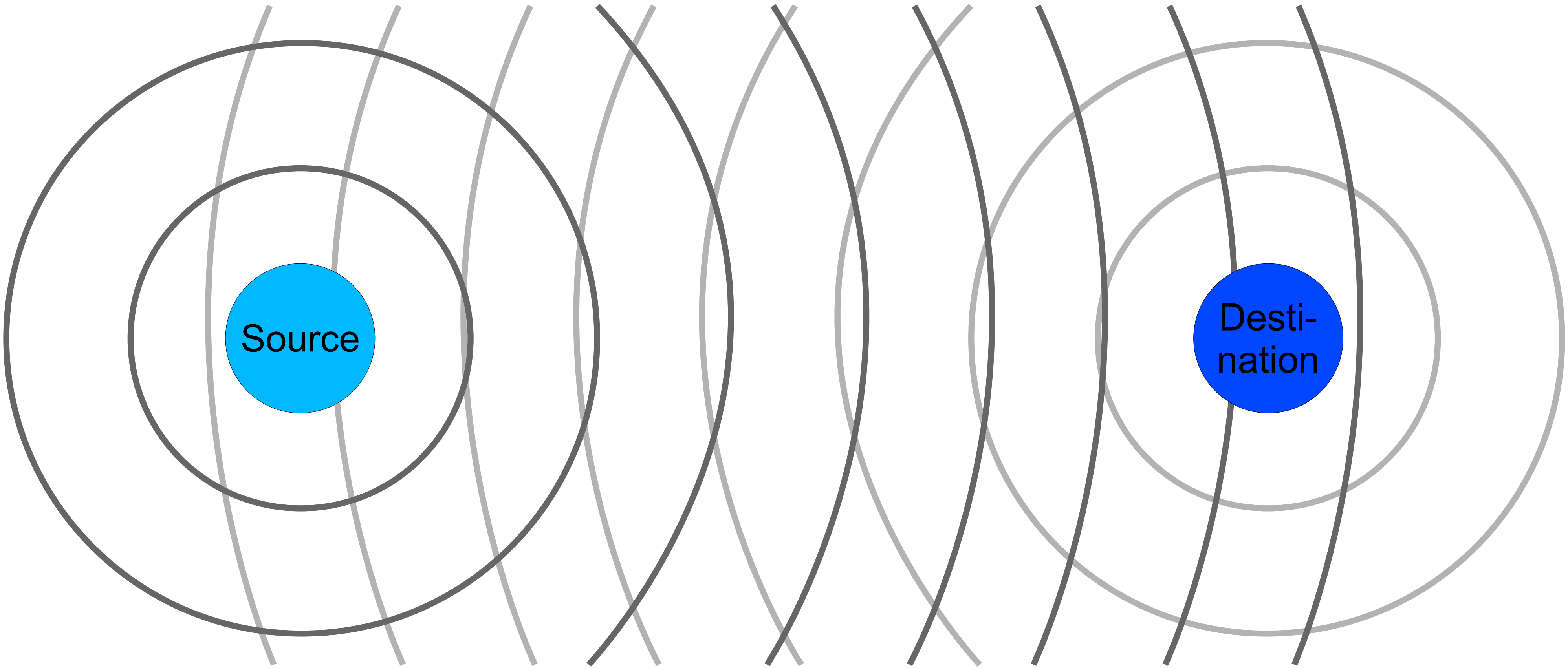
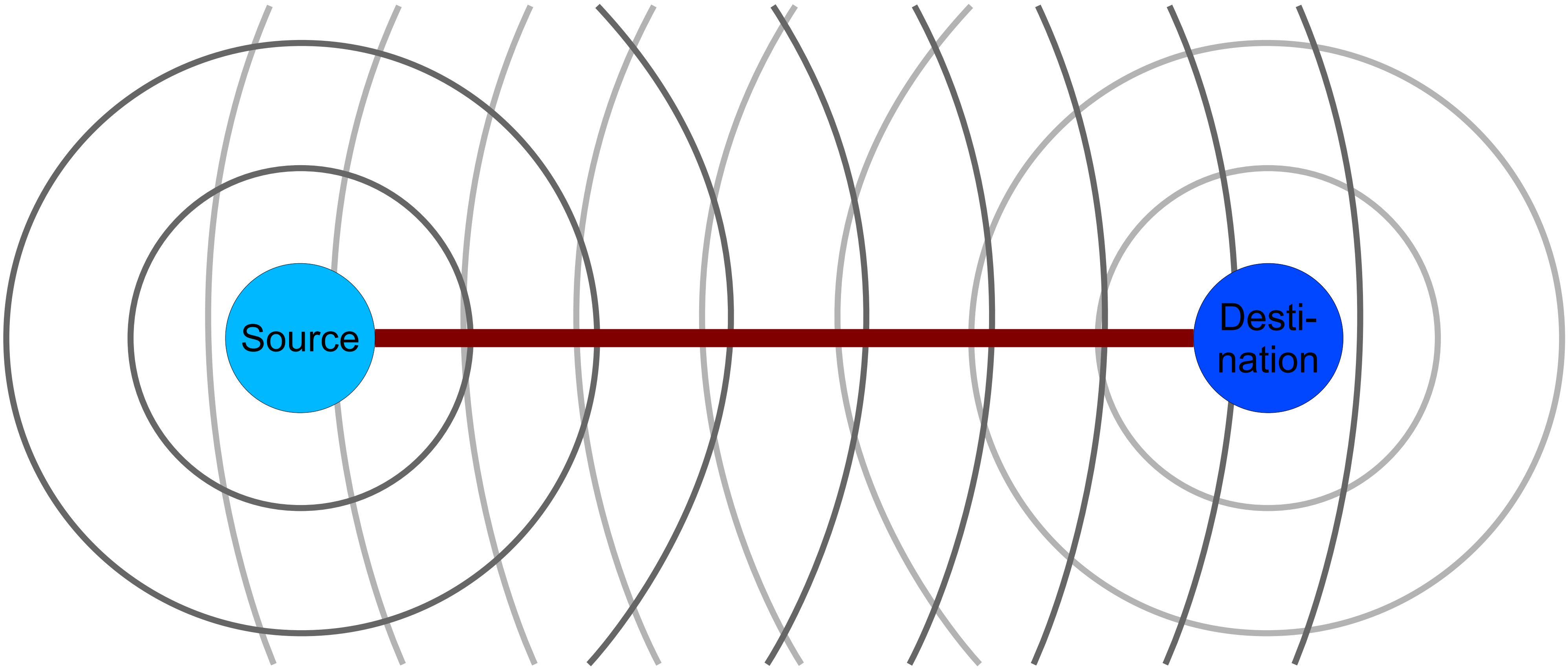


(cf. Butera)

Geometric Program: Channel



(cf. Butera)

Geometric Program: Channel

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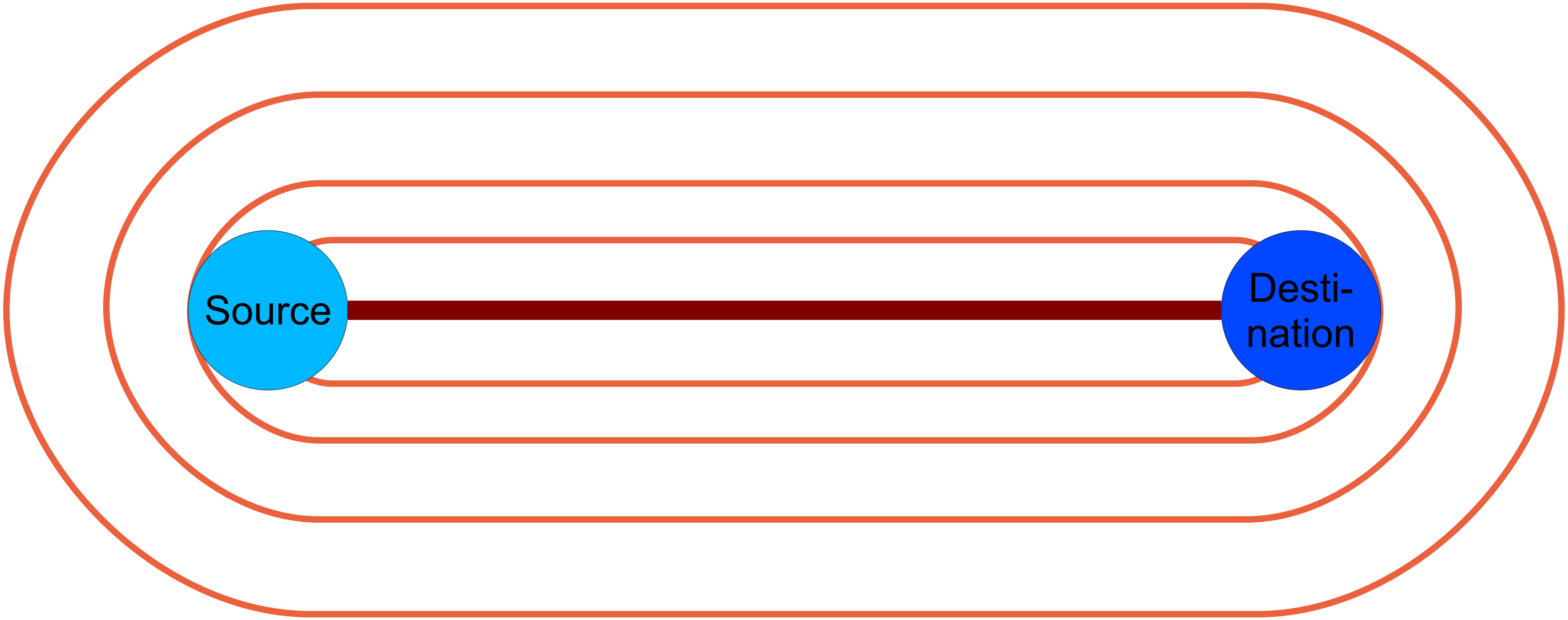
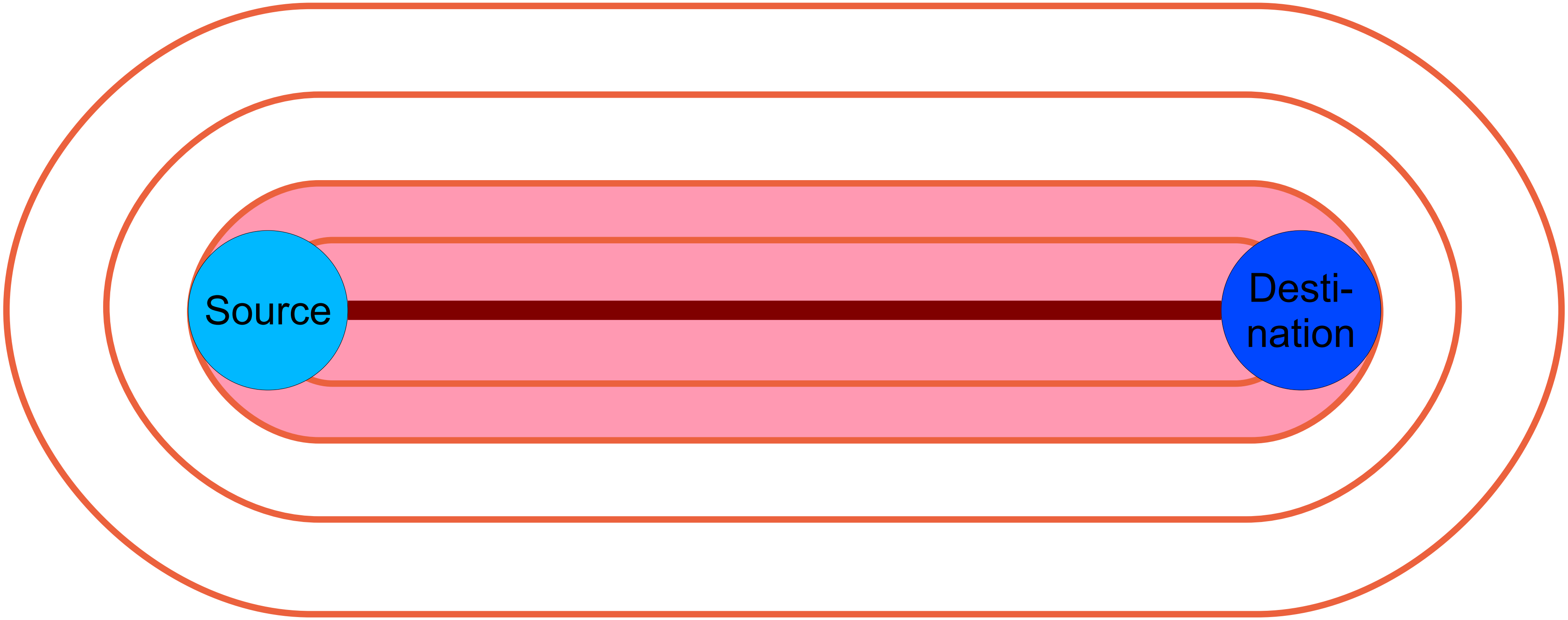


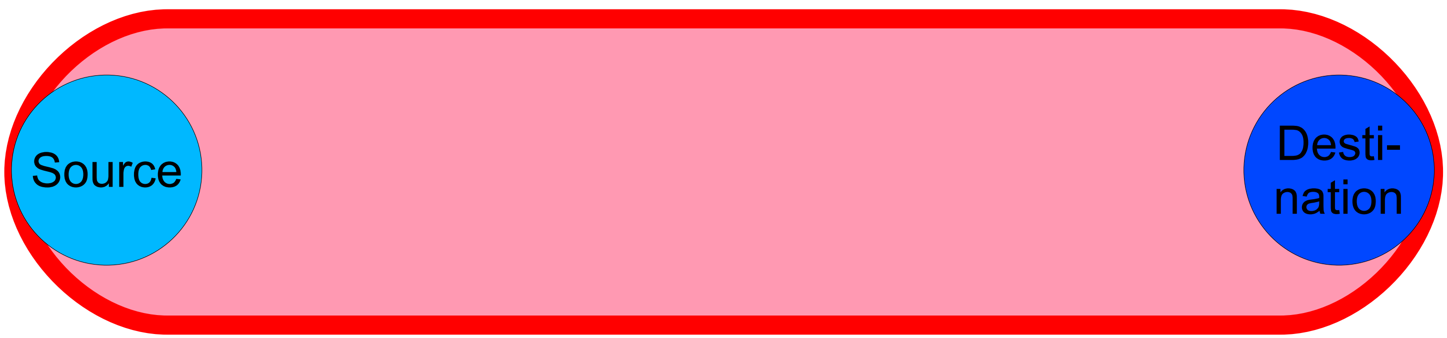
(cf. Butera)

Geometric Program: Channel



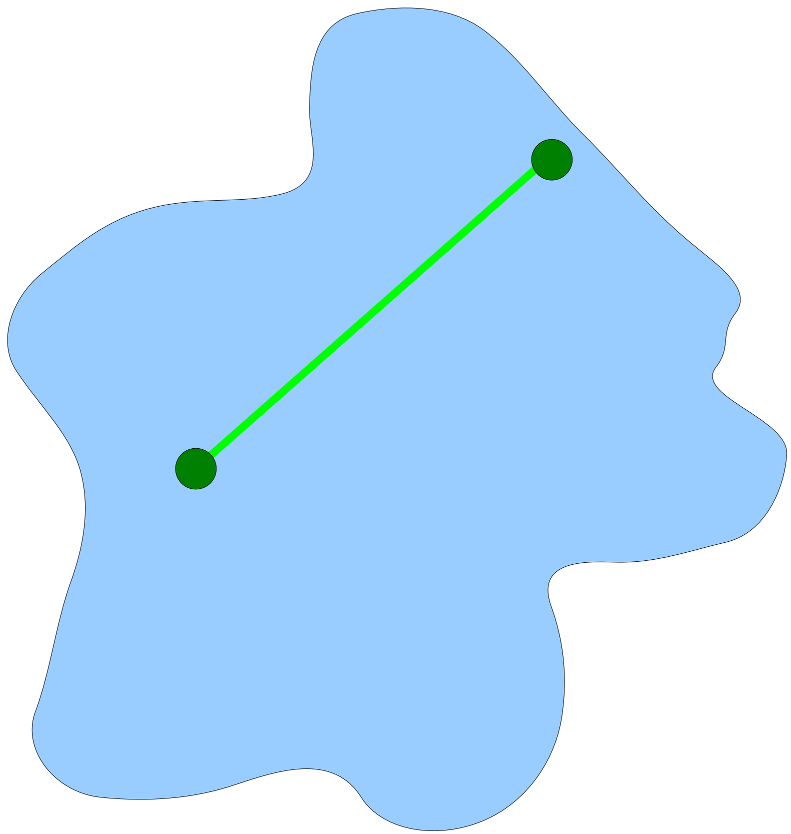
(cf. Butera)

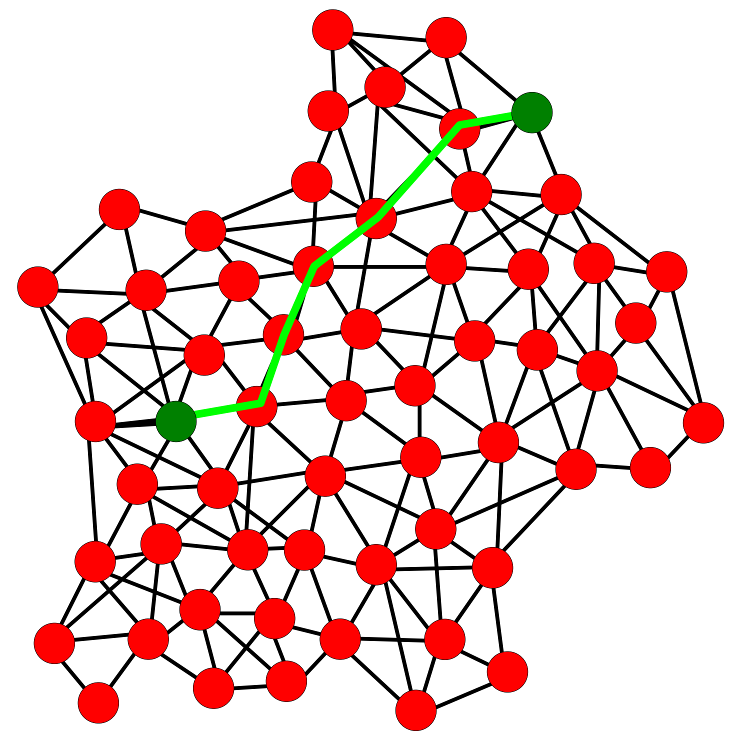
Geometric Program: Channel



(cf. Butera)

Discretization Error

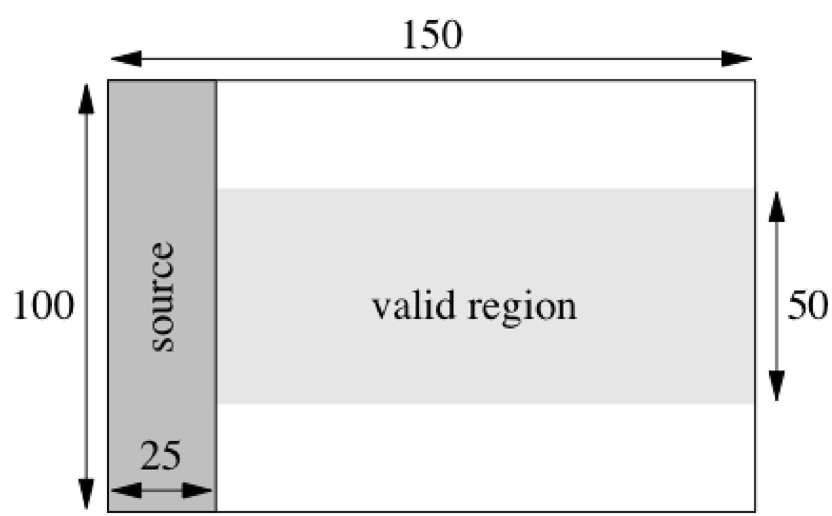




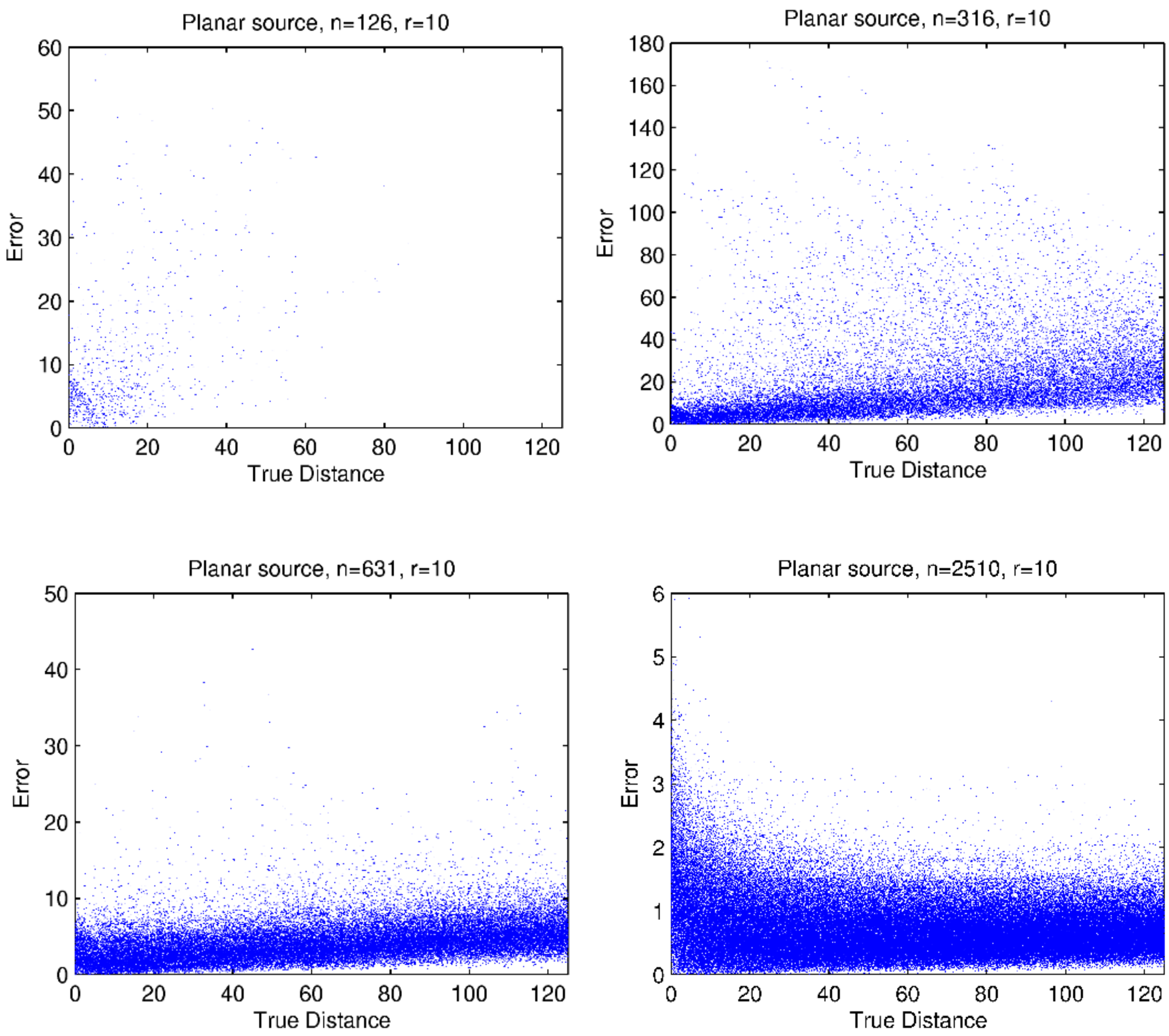
Prediction: $\varepsilon = \alpha\rho^{-2}d$

Experimental Strategy

- Distribute *n* devices randomly in area *A*, communicating in *r* range, for density *ρ*
- Perfect range information, no failures
- Survey wide range of parameters
  - 100 trials/combination, ~20K total



Four Domains of Behavior



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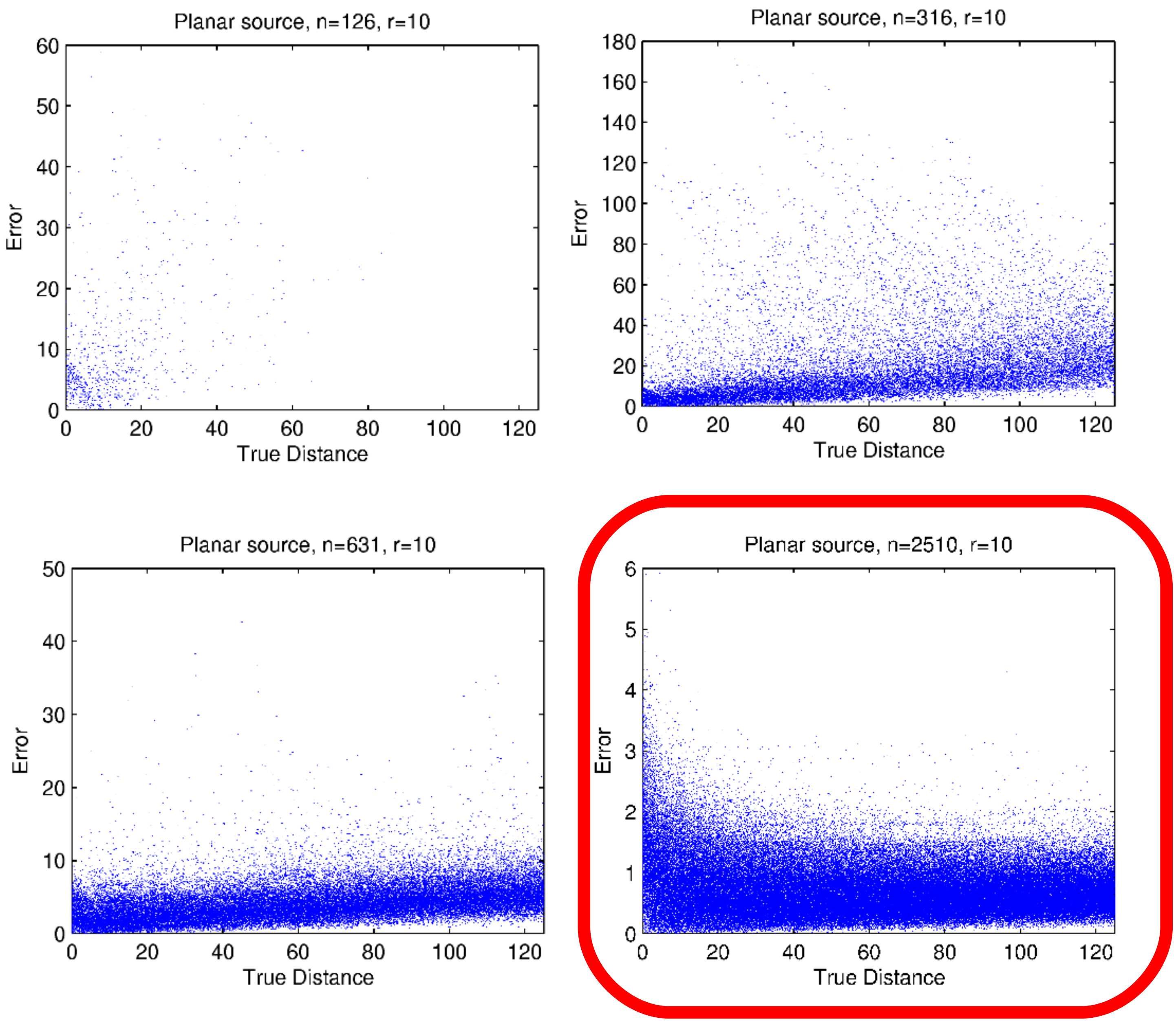
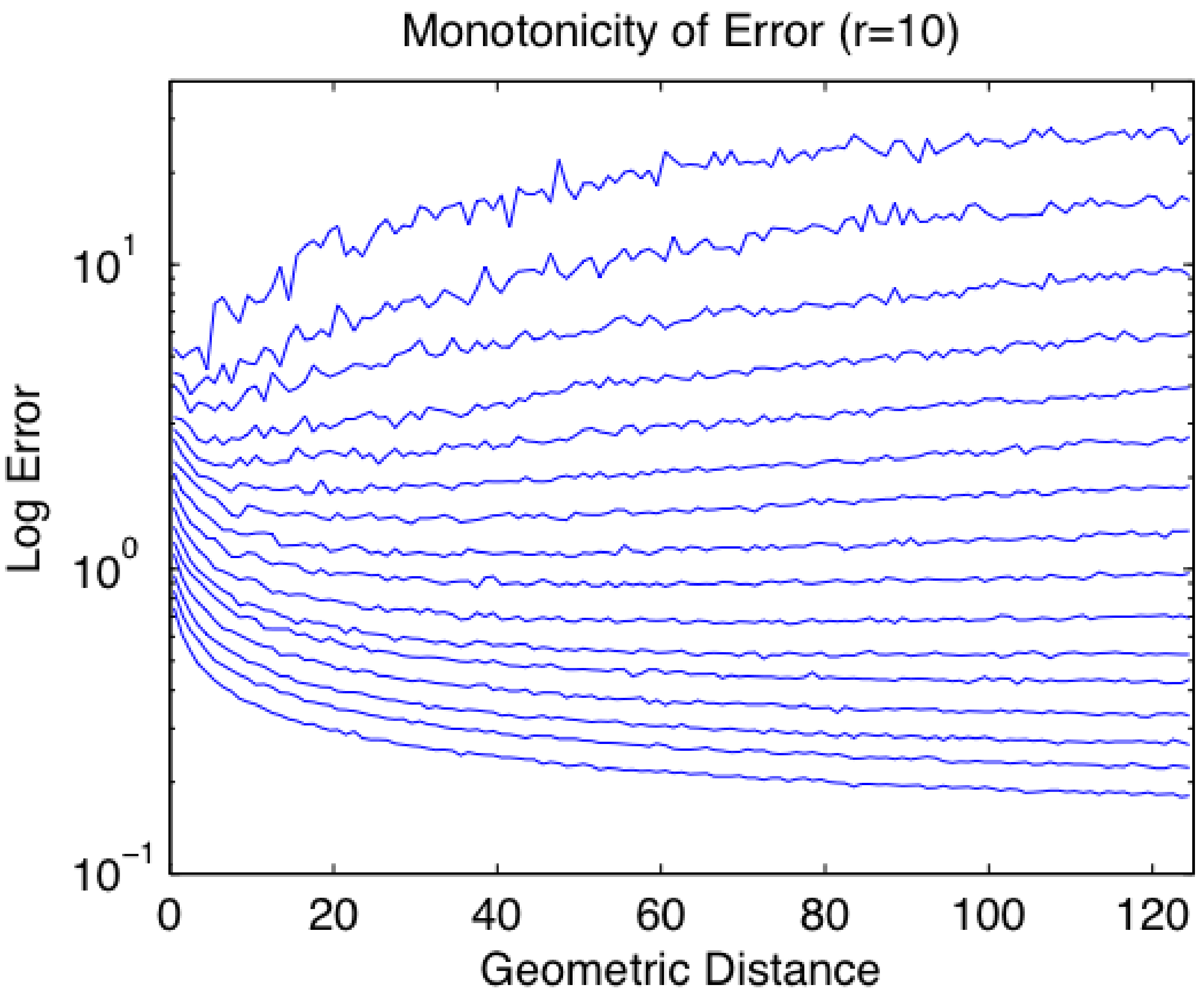


Density affects error monotonically



Density = 6.6 to 209.4

Making an Empirical Model

$$\bar{\varepsilon_G} = \alpha d + \beta d^{-\gamma}$$

$$\bar{\varepsilon_G} = \alpha_1 \rho^{\alpha_2} d + \beta_1 \rho^{\beta_2} d^{(\gamma_1 + \gamma_2 \rho^{\gamma_3})}$$

Name	Value	95% confidence bounds
α_1	7.8	(6.8, 8.7)
α_2	-2.14	(-2.19, -2.10)
β_1	11.2	(10.8, 11.5)
β_2	-0.516	(-0.526, -0.505)
γ_1	-0.292	(-0.303, -0.282)
γ_2	1.6	(1.3, 1.9)
γ_3	-0.77	(-0.86, -0.69)

Mean

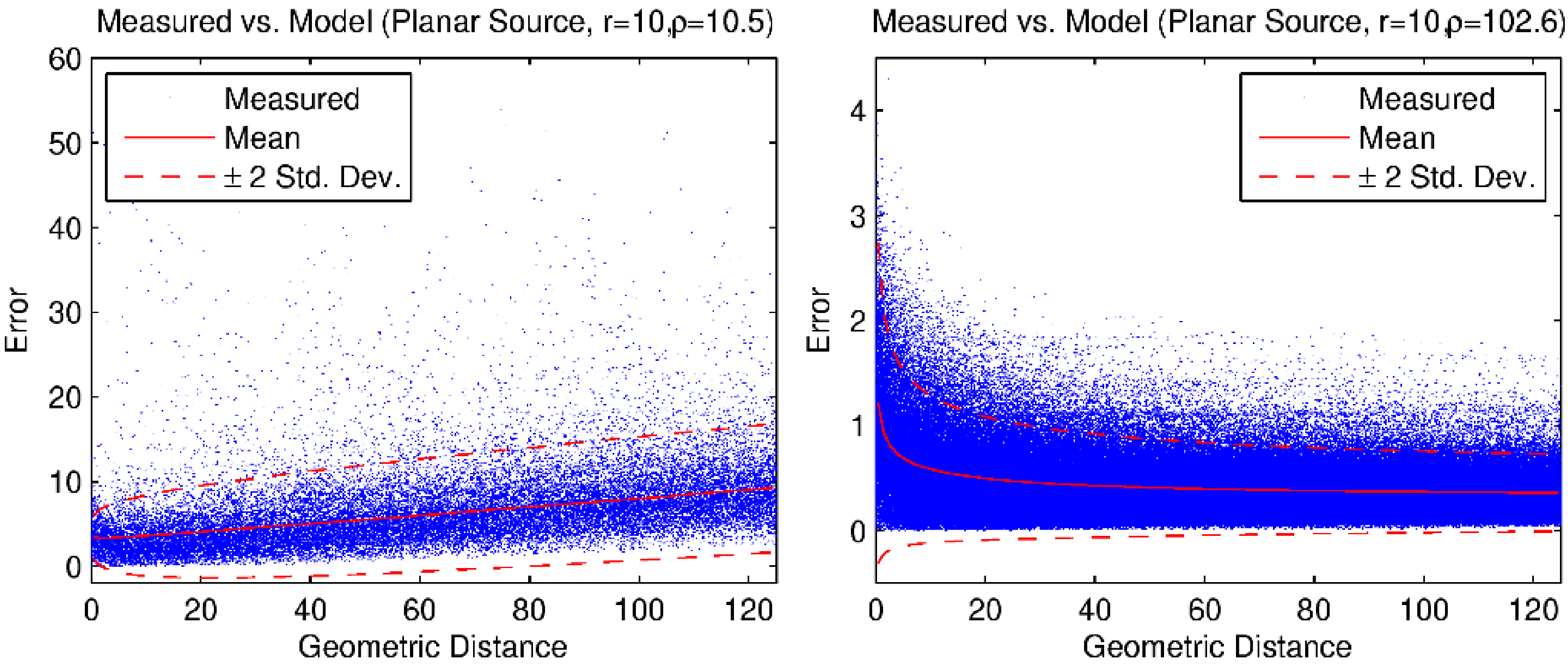
$$\sigma_{\varepsilon_G} = \kappa + \lambda d^{-\mu}$$

$$\sigma_{\varepsilon_G} = \kappa_1 \rho^{\kappa_2} + \lambda_1 \rho^{\lambda_2} d^{(\mu_1 + \mu_2 \rho^{\mu_3})}$$

Name	Value	95% confidence bounds
κ_1	-25000	(-52000, 2000)
κ_2	-4.5	(-4.9, -4.0)
λ_1	7.40	(7.07, 7.73)
λ_2	-0.529	(-0.541, -0.517)
μ_1	-0.278	(-0.283, -0.272)
μ_2	11	(5, 16)
μ_3	-1.38	(-1.54, -1.21)

Standard Deviation

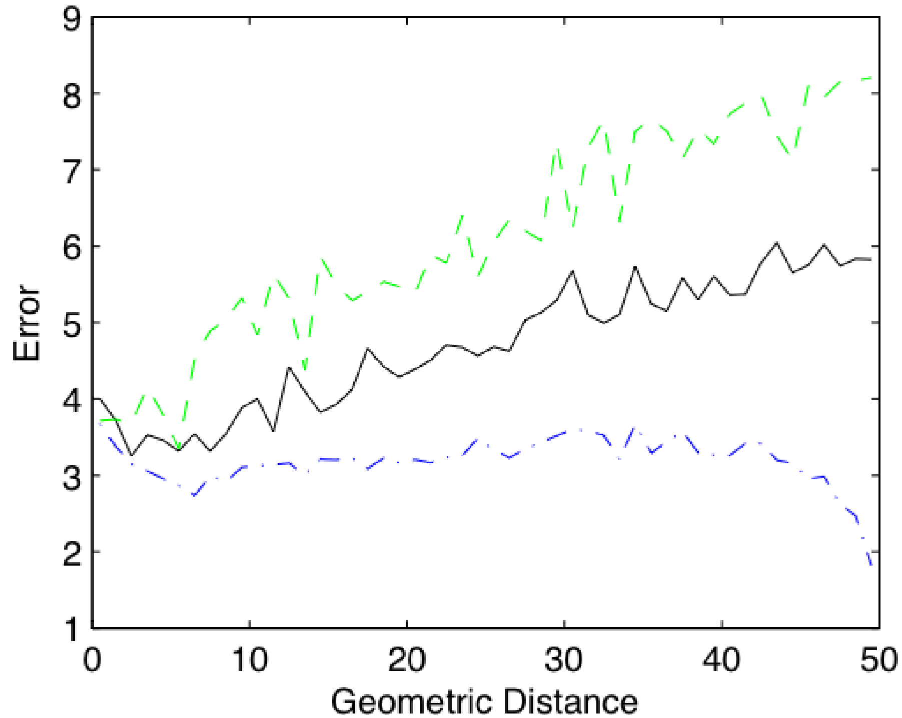
Model Fit



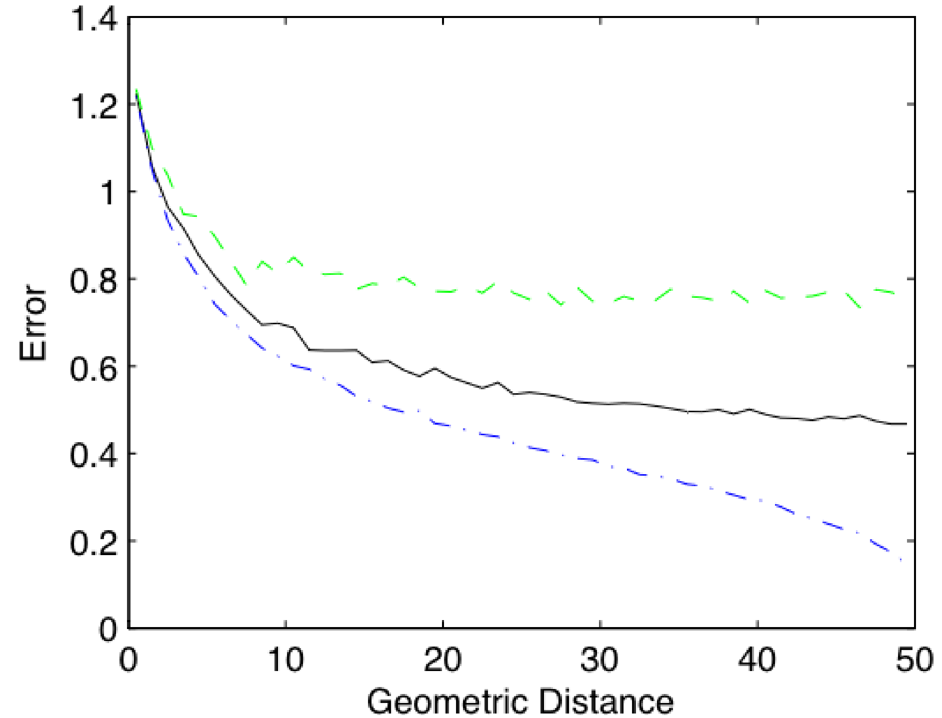
Measured vs. Model (Planar Source, r=10,ρ=10.5)
Measured
Mean
± 2 Std. Dev.
Error
Geometric Distance
0
10
20
30
40
50
60
0
20
40
60
80
100
120
Measured vs. Model (Planar Source, r=10,ρ=102.6)
Measured
Mean
± 2 Std. Dev.
Error
Geometric Distance
0
1
2
3
4
0
20
40
60
80
100
120

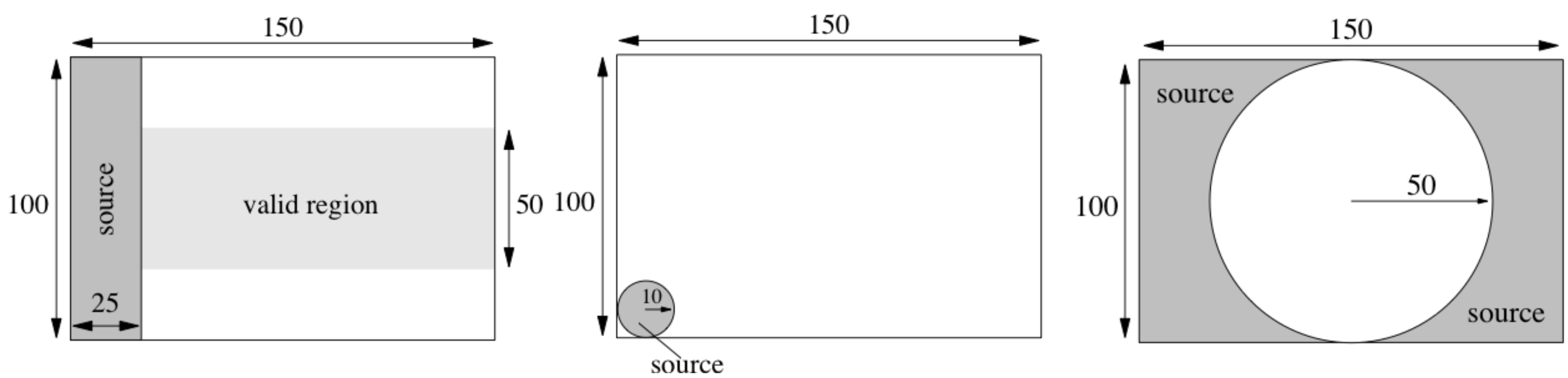
Source shape matters

density=10.4

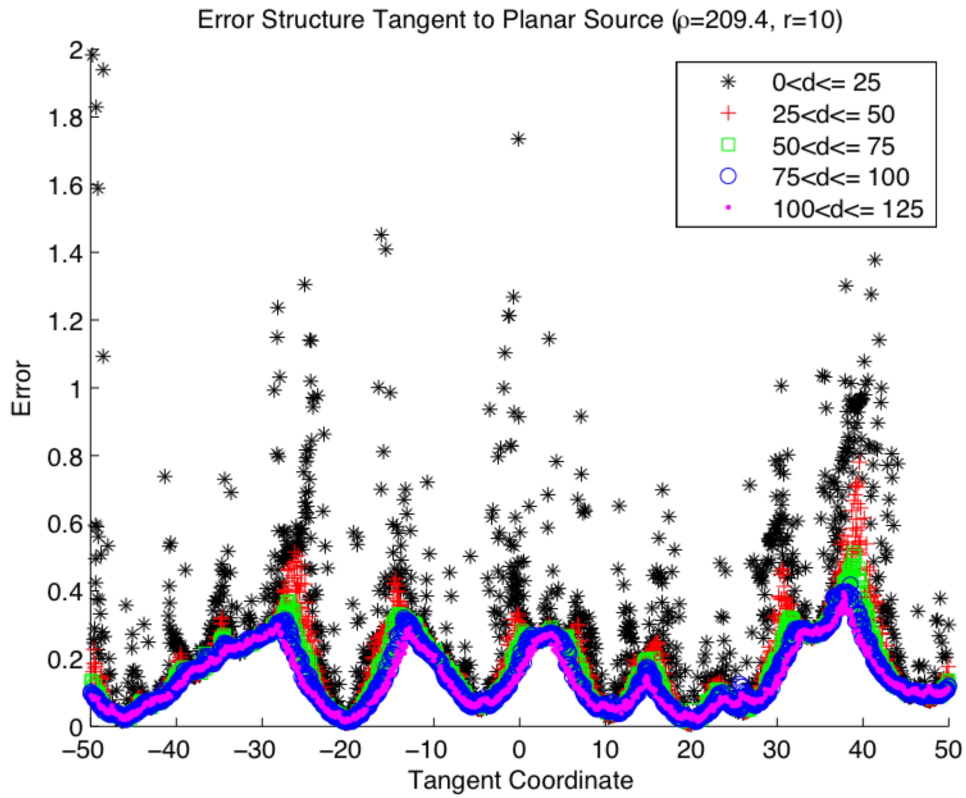


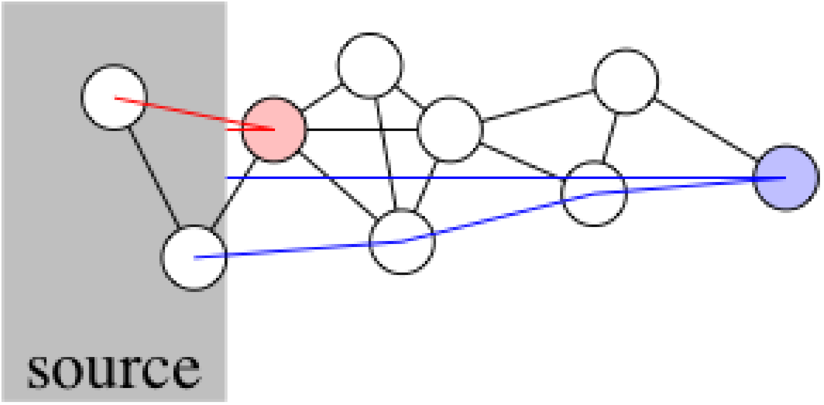
density=83.4



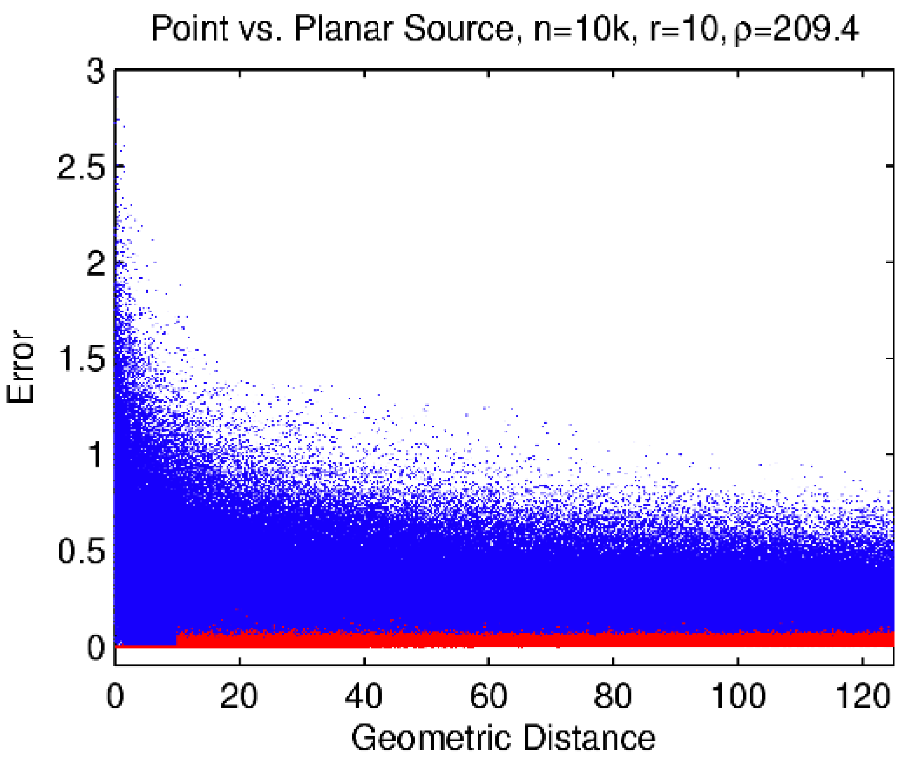


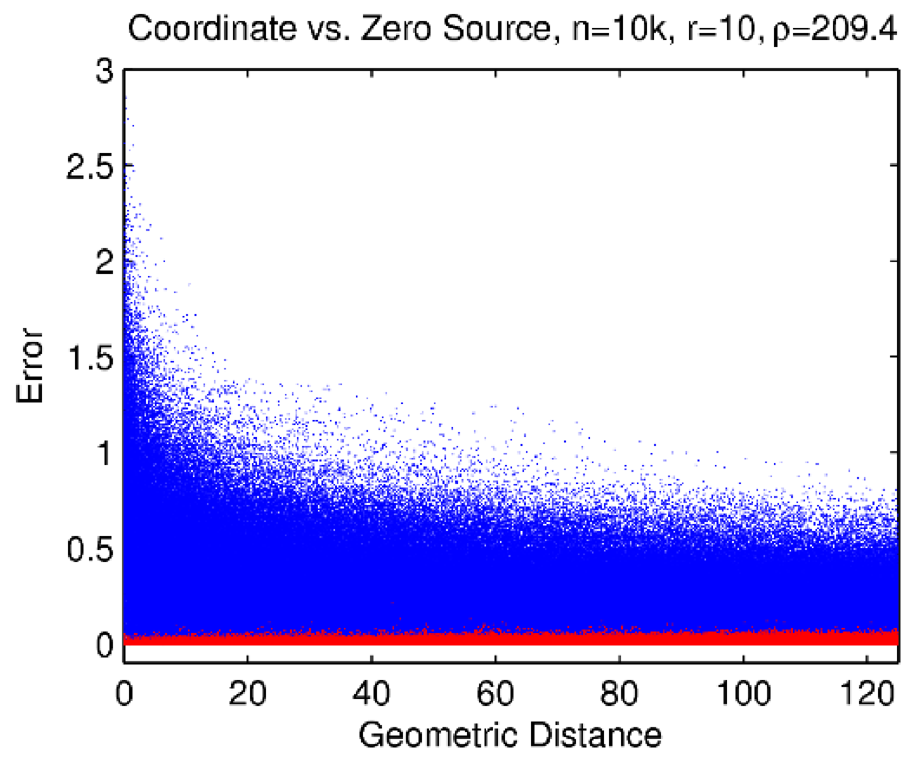
Understanding the Transient





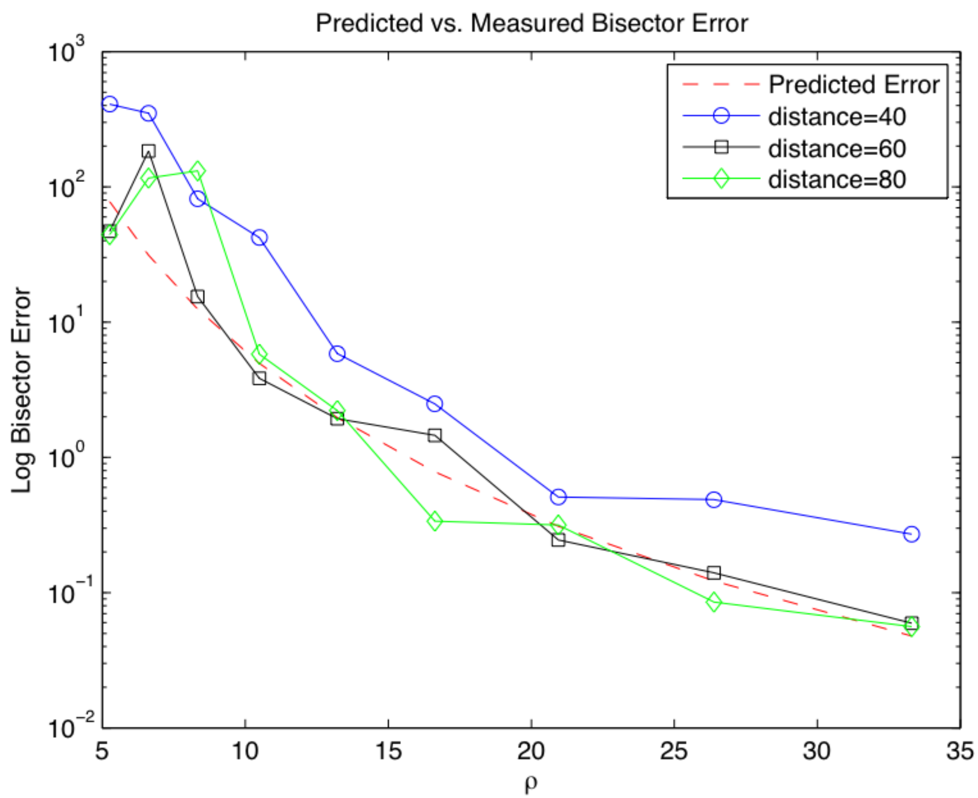
Transient Elimination

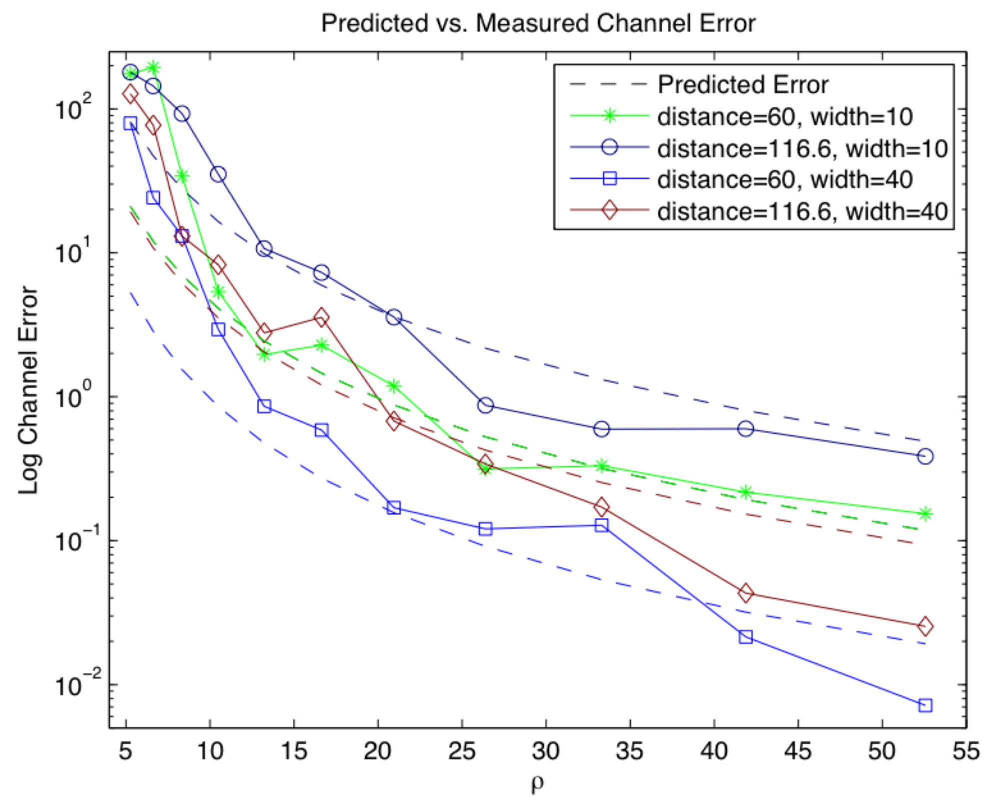




Point or “true depth” sources eliminate transient

Model Predicts Channel/Bisector





Contributions

- Identified new gradient phenomena
- Created empirical model of gradient error
- Used model to predict channel & bisector error
- Laid foundation for better theoretical prediction of composed gradient programs